

# NAG Toolbox for MATLAB

## s20ad

### 1 Purpose

s20ad returns a value for the Fresnel Integral  $C(x)$ , via the function name.

### 2 Syntax

```
[result, ifail] = s20ad(x)
```

### 3 Description

s20ad evaluates an approximation to the Fresnel Integral

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) dt.$$

**Note:**  $C(x) = -C(-x)$ , so the approximation need only consider  $x \geq 0.0$ .

The function is based on three Chebyshev expansions:

For  $0 < x \leq 3$ ,

$$C(x) = x \sum_{r=0}' a_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{3}\right)^4 - 1.$$

For  $x > 3$ ,

$$C(x) = \frac{1}{2} + \frac{f(x)}{x} \sin\left(\frac{\pi}{2}x^2\right) - \frac{g(x)}{x^3} \cos\left(\frac{\pi}{2}x^2\right),$$

where  $f(x) = \sum_{r=0}' b_r T_r(t)$ ,

and  $g(x) = \sum_{r=0}' c_r T_r(t)$ ,

with  $t = 2\left(\frac{3}{x}\right)^4 - 1$ .

For small  $x$ ,  $C(x) \simeq x$ . This approximation is used when  $x$  is sufficiently small for the result to be correct to **machine precision**.

For large  $x$ ,  $f(x) \simeq \frac{1}{\pi}$  and  $g(x) \simeq \frac{1}{\pi^2}$ . Therefore for moderately large  $x$ , when  $\frac{1}{\pi^2 x^3}$  is negligible compared with  $\frac{1}{2}$ , the second term in the approximation for  $x > 3$  may be dropped. For very large  $x$ , when  $\frac{1}{\pi x}$  becomes negligible,  $C(x) \simeq \frac{1}{2}$ . However there will be considerable difficulties in calculating  $\sin\left(\frac{\pi}{2}x^2\right)$  accurately before this final limiting value can be used. Since  $\sin\left(\frac{\pi}{2}x^2\right)$  is periodic, its value is essentially determined by the fractional part of  $x^2$ . If  $x^2 = N + \theta$ , where  $N$  is an integer and  $0 \leq \theta < 1$ , then  $\sin\left(\frac{\pi}{2}x^2\right)$  depends on  $\theta$  and on  $N$  modulo 4. By exploiting this fact, it is possible to retain some significance in the calculation of  $\sin\left(\frac{\pi}{2}x^2\right)$  either all the way to the very large  $x$  limit, or at least until the integer part of  $\frac{x}{2}$  is equal to the maximum integer allowed on the machine.

## 4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument  $x$  of the function.

### 5.2 Optional Input Parameters

None.

### 5.3 Input Parameters Omitted from the MATLAB Interface

None.

### 5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

There are no failure exits from s20ad. The parameter **ifail** has been included for consistency with other functions in this chapter.

## 7 Accuracy

Let  $\delta$  and  $\epsilon$  be the relative errors in the argument and result respectively.

If  $\delta$  is somewhat larger than the *machine precision* (i.e if  $\delta$  is due to data errors etc.), then  $\epsilon$  and  $\delta$  are approximately related by:

$$\epsilon \simeq \left| \frac{x \cos\left(\frac{\pi}{2}x^2\right)}{C(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor  $\left| \frac{x \cos\left(\frac{\pi}{2}x^2\right)}{C(x)} \right|$ .

However, if  $\delta$  is of the same order as the *machine precision*, then rounding errors could make  $\epsilon$  slightly larger than the above relation predicts.

For small  $x$ ,  $\epsilon \simeq \delta$  and there is no amplification of relative error.

For moderately large values of  $x$ ,

$$|\epsilon| \simeq \left| 2x \cos\left(\frac{\pi}{2}x^2\right) \right| |\delta|$$

and the result will be subject to increasingly large amplification of errors. However the above relation

breaks down for large values of  $x$  (i.e., when  $\frac{1}{x^2}$  is of the order of the *machine precision*); in this region the relative error in the result is essentially bounded by  $\frac{2}{\pi x}$ .

Hence the effects of error amplification are limited and at worst the relative error loss should not exceed half the possible number of significant figures.

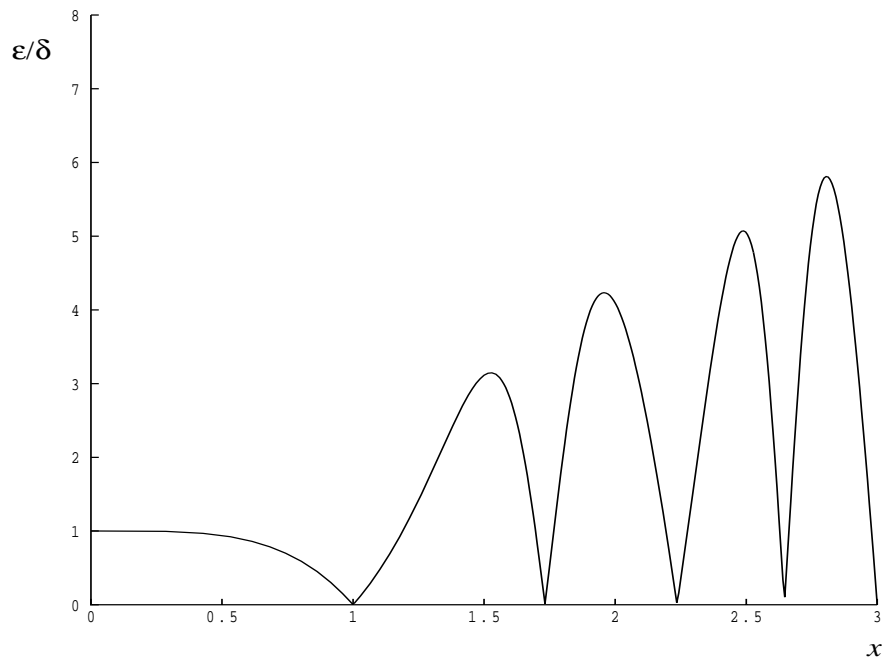


Figure 1

## 8 Further Comments

None.

## 9 Example

```
x = 0;
[result, ifail] = s20ad(x)

result =
      0
ifail =
      0
```